PG No. 11 C_{4h} $4/m$ [tetragonal] (axial, internal polar quadrupole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(2,1)}[q](A_g)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{30}Q_v xy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(2,1)}[q](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(2,1)}[q](E_g)$

** symmetry

$$y$$

$$x$$

** expression

$$\frac{3\sqrt{10}Q_u xz}{10} - \frac{\sqrt{30}Q_v xz}{10} - \frac{\sqrt{30}Q_{xy}yz}{10} + \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$-\frac{3\sqrt{10}Q_u yz}{10} - \frac{\sqrt{30}Q_v yz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(2,-1)}[q](A_u)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$\vec{\mathbb{G}}_2^{(2,1)}[q](A_u)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}Q_v xyz}{14} - \frac{5\sqrt{42}Q_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xz}y\left(x^2+y^2-4z^2\right)}{28} + \frac{\sqrt{42}Q_{yz}x\left(x^2+y^2-4z^2\right)}{28}$$

$\vec{\mathbb{G}}_2^{(2,-1)}[q](B_u, 1)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$\vec{\mathbb{G}}_2^{(2,-1)}[q](B_u, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{6}Q_v z}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$\vec{\mathbb{G}}_2^{(2,1)}[q](B_u, 1)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$-\frac{5\sqrt{42}Q_{u}xyz}{14}+\frac{\sqrt{14}Q_{xy}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{28}-\frac{\sqrt{14}Q_{xz}y\left(9x^{2}-y^{2}-6z^{2}\right)}{28}+\frac{\sqrt{14}Q_{yz}x\left(x^{2}-9y^{2}+6z^{2}\right)}{28}$$

$\vec{\mathbb{G}}_{2}^{(2,1)}[q](B_u,2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{5\sqrt{42}Q_{u}z\left(x-y\right)\left(x+y\right)}{28}-\frac{\sqrt{14}Q_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{28}+\frac{\sqrt{14}Q_{xz}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14}+\frac{\sqrt{14}Q_{yz}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E_u)$, $\vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E_u)$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{2}Q_{u}y}{2}+\frac{\sqrt{6}Q_{v}y}{6}-\frac{\sqrt{6}Q_{xy}x}{6}+\frac{\sqrt{6}Q_{yz}z}{6}$$

$$\frac{\sqrt{2}Q_{u}x}{2}+\frac{\sqrt{6}Q_{v}x}{6}+\frac{\sqrt{6}Q_{xy}y}{6}-\frac{\sqrt{6}Q_{xz}z}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E_u)$, $\vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E_u)$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{42}Q_{u}y\left(x^{2}+y^{2}-4z^{2}\right)}{28}+\frac{\sqrt{14}Q_{v}y\left(9x^{2}-y^{2}-6z^{2}\right)}{28}-\frac{\sqrt{14}Q_{xy}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14}-\frac{\sqrt{14}Q_{yz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$$-\frac{\sqrt{42}Q_{u}x\left(x^{2}+y^{2}-4z^{2}\right)}{28}-\frac{\sqrt{14}Q_{v}x\left(x^{2}-9y^{2}+6z^{2}\right)}{28}-\frac{\sqrt{14}Q_{xy}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14}+\frac{\sqrt{14}Q_{xz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](A_g)$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{30}Q_{v}xy}{10}+\frac{\sqrt{30}Q_{xy}\left(x-y\right)\left(x+y\right)}{20}+\frac{\sqrt{30}Q_{xz}yz}{5}-\frac{\sqrt{30}Q_{yz}xz}{5}$$

$\vec{\mathbb{G}}_{3}^{(2,1)}[q](A_g)$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_{v}xy\left(x^{2}+y^{2}-6z^{2}\right)}{6}+\frac{\sqrt{15}Q_{xy}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{12}-\frac{\sqrt{15}Q_{xz}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}+\frac{\sqrt{15}Q_{yz}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}$$

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](B_g,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}Q_{u}\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{2}Q_{v}\left(x^{2}+y^{2}-2z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_3^{(2,-1)}[q](B_g,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{6}Q_{u}xy}{2}-\frac{\sqrt{2}Q_{xy}\left(x^{2}+y^{2}-2z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_3^{(2,1)}[q](B_g,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}Q_{u}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{6}-\frac{Q_{v}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{6}-\frac{7Q_{xy}xy\left(x-y\right)\left(x+y\right)}{6}+\frac{7Q_{xz}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{7Q_{yz}yz\left(y-z\right)\left(y+z\right)}{6}$$

$\vec{\mathbb{G}}_3^{(2,1)}[q](B_g,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{u}xy\left(x^{2}+y^{2}-6z^{2}\right)}{3}+\frac{7Q_{v}xy\left(x-y\right)\left(x+y\right)}{6}-\frac{Q_{xy}\left(5x^{4}-18x^{2}y^{2}-12x^{2}z^{2}+5y^{4}-12y^{2}z^{2}+4z^{4}\right)}{12}-\frac{7Q_{xz}yz\left(3x^{2}+y^{2}-2z^{2}\right)}{12}-\frac{7Q_{yz}xz\left(x^{2}+3y^{2}-2z^{2}\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E_g,1)$
** symmetry

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

** expression

$$-\frac{3\sqrt{10}Q_{u}xz}{20}+\frac{\sqrt{30}Q_{v}xz}{20}-\frac{\sqrt{30}Q_{xy}yz}{5}-\frac{\sqrt{30}Q_{xz}\left(x-z\right)\left(x+z\right)}{20}+\frac{\sqrt{30}Q_{yz}xy}{5}$$

$$\frac{3\sqrt{10}Q_{u}yz}{20}+\frac{\sqrt{30}Q_{v}yz}{20}+\frac{\sqrt{30}Q_{xy}xz}{5}-\frac{\sqrt{30}Q_{xz}xy}{5}+\frac{\sqrt{30}Q_{yz}\left(y-z\right)\left(y+z\right)}{20}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{u}yz}{4}-\frac{3\sqrt{2}Q_{v}yz}{4}+\frac{\sqrt{2}Q_{yz}\left(2x^{2}-y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{6}Q_{u}xz}{4}+\frac{3\sqrt{2}Q_{v}xz}{4}-\frac{\sqrt{2}Q_{xz}\left(x^{2}-2y^{2}+z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E_g,1)$
** symmetry

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_u xz\left(x^2-6y^2+z^2\right)}{4}+\frac{\sqrt{15}Q_v xz\left(x^2-6y^2+z^2\right)}{12}+\frac{\sqrt{15}Q_{xy}yz\left(3x^2-4y^2+3z^2\right)}{12}-\frac{\sqrt{15}Q_{xz}\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{12}-\frac{\sqrt{15}Q_{yz}xy\left(3x^2-4y^2+3z^2\right)}{12}$$

$$-\frac{\sqrt{5}Q_u yz\left(6x^2-y^2-z^2\right)}{4}-\frac{\sqrt{15}Q_v yz\left(6x^2-y^2-z^2\right)}{12}+\frac{\sqrt{15}Q_{xy}xz\left(4x^2-3y^2-3z^2\right)}{12}-\frac{\sqrt{15}Q_{xz}xy\left(4x^2-3y^2-3z^2\right)}{12}-\frac{\sqrt{15}Q_{yz}\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E_g,2)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_u yz\left(12x^2-9y^2+5z^2\right)}{12}-\frac{Q_v yz\left(36x^2+y^2-13z^2\right)}{12}+\frac{7Q_{xy}xz\left(2x^2-3y^2-z^2\right)}{12}+\frac{7Q_{xz}xy\left(2x^2-y^2-3z^2\right)}{12}-\frac{Q_{yz}\left(4x^4-12x^2y^2-12x^2z^2+5y^4-18y^2z^2+5z^4\right)}{12}$$

$$-\frac{\sqrt{3}Q_u xz\left(9x^2-12y^2-5z^2\right)}{12}+\frac{Q_v xz\left(x^2+36y^2-13z^2\right)}{12}-\frac{7Q_{xy}yz\left(3x^2-2y^2+z^2\right)}{12}-\frac{Q_{xz}\left(5x^4-12x^2y^2-18x^2z^2+4y^4-12y^2z^2+5z^4\right)}{12}-\frac{7Q_{yz}xy\left(x^2-2y^2+3z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{5}Q_{xy}z\left(x-y\right)\left(x+y\right)}{2}-\frac{\sqrt{5}Q_{xz}y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_{yz}x\left(y-z\right)\left(y+z\right)}{2}$$

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_u,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{6\sqrt{7}Q_v xyz}{7}-\frac{\sqrt{7}Q_{xy}z\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{7}Q_{xz}y\left(4x^2-3y^2+5z^2\right)}{14}+\frac{\sqrt{7}Q_{yz}x\left(3x^2-4y^2-5z^2\right)}{14}$$

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_u,3)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_v z\left(x-y\right)\left(x+y\right)}{2}+\sqrt{3}Q_{xy}xyz+\frac{\sqrt{3}Q_{xz}x\left(x^2-3y^2\right)}{4}-\frac{\sqrt{3}Q_{yz}y\left(3x^2-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4}^{(2,1)}[q](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{3\sqrt{2310}Q_u xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{3\sqrt{770}Q_v xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{770}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{22}-\frac{\sqrt{770}Q_{xz}y\left(x-z\right)\left(x+z\right)\left(x^2-2y^2+z^2\right)}{22}-\frac{\sqrt{770}Q_{yz}x\left(y-z\right)\left(y+z\right)\left(2x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_u,2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$\frac{21\sqrt{66}Q_u xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{21\sqrt{22}Q_v xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{7\sqrt{22}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{22}Q_{xz}y\left(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4\right)}{44}-\frac{\sqrt{22}Q_{yz}x\left(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4\right)}{44}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_u,3)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{9\sqrt{154}Q_u z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}-\frac{\sqrt{462}Q_v z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{462}Q_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{462}Q_{xz}x\left(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2\right)}{44}+\frac{\sqrt{462}Q_{yz}y\left(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_u,1)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{6\sqrt{7}Q_u xyz}{7}+\frac{\sqrt{21}Q_{xy}z\left(3x^2+3y^2-2z^2\right)}{14}-\frac{\sqrt{21}Q_{xz}y\left(2x^2-y^2+z^2\right)}{14}+\frac{\sqrt{21}Q_{yz}x\left(x^2-2y^2-z^2\right)}{14}$$

$\vec{\mathbb{G}}_4^{(2,-1)}[q](B_u,2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_u z\left(x-y\right)\left(x+y\right)}{7}+\frac{\sqrt{21}Q_v z\left(3x^2+3y^2-2z^2\right)}{14}-\frac{\sqrt{21}Q_{xz}x\left(x^2-5y^2+2z^2\right)}{28}-\frac{\sqrt{21}Q_{yz}y\left(5x^2-y^2-2z^2\right)}{28}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B_u,1)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{22}Q_u xyz\left(x^2+y^2-2z^2\right)}{44}-\frac{21\sqrt{66}Q_v xyz\left(x-y\right)\left(x+y\right)}{44}+\frac{\sqrt{66}Q_{xy}z\left(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4\right)}{44}-\frac{\sqrt{66}Q_{xz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}-\frac{\sqrt{66}Q_{yz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](B_u,2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{21\sqrt{22}Q_u z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}-\frac{\sqrt{66}Q_v z\left(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4\right)}{88}-\frac{21\sqrt{66}Q_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{66}Q_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E_u,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{3Q_{u}x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}Q_{v}x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}Q_{xy}y\left(y^{2}-3z^{2}\right)}{4}-\frac{\sqrt{3}Q_{xz}z\left(3y^{2}-z^{2}\right)}{4}+\sqrt{3}Q_{yz}xyz$$

$$\frac{3Q_{u}y\left(x-z\right)\left(x+z\right)}{4}-\frac{\sqrt{3}Q_{v}y\left(x-z\right)\left(x+z\right)}{4}+\frac{\sqrt{3}Q_{xy}x\left(x^{2}-3z^{2}\right)}{4}+\sqrt{3}Q_{xz}xyz-\frac{\sqrt{3}Q_{yz}z\left(3x^{2}-z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E_u,2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_{u}y\left(5x^{2}-2y^{2}+z^{2}\right)}{28}+\frac{\sqrt{21}Q_{v}y\left(3x^{2}+2y^{2}-9z^{2}\right)}{28}+\frac{\sqrt{21}Q_{xy}x\left(x^{2}+2y^{2}-5z^{2}\right)}{28}+\frac{\sqrt{21}Q_{yz}z\left(5x^{2}-2y^{2}-z^{2}\right)}{28}$$

$$\frac{3\sqrt{7}Q_{u}x\left(2x^{2}-5y^{2}-z^{2}\right)}{28}+\frac{\sqrt{21}Q_{v}x\left(2x^{2}+3y^{2}-9z^{2}\right)}{28}-\frac{\sqrt{21}Q_{xy}y\left(2x^{2}+y^{2}-5z^{2}\right)}{28}+\frac{\sqrt{21}Q_{xz}z\left(2x^{2}-5y^{2}+z^{2}\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E_u,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{3\sqrt{154}Q_{u}x\left(x^{2}y^{2}-x^{2}z^{2}+y^{4}-9y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{462}Q_{v}x\left(x^{2}y^{2}-x^{2}z^{2}-5y^{4}+27y^{2}z^{2}-4z^{4}\right)}{88}-\frac{\sqrt{462}Q_{xy}y\left(2x^{2}y^{2}-6x^{2}z^{2}-y^{4}+8y^{2}z^{2}-3z^{4}\right)}{44}+\frac{\sqrt{462}Q_{xz}z\left(6x^{2}y^{2}-2x^{2}z^{2}+3y^{4}-8y^{2}z^{2}+z^{4}\right)}{44}-\frac{\sqrt{462}Q_{yz}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{44}$$

$$-\frac{3\sqrt{154}Q_{u}y\left(x^{4}+x^{2}y^{2}-9x^{2}z^{2}-y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{462}Q_{v}y\left(5x^{4}-x^{2}y^{2}-27x^{2}z^{2}+y^{2}z^{2}+4z^{4}\right)}{88}+\frac{\sqrt{462}Q_{xy}x\left(x^{4}-2x^{2}y^{2}-8x^{2}z^{2}+6y^{2}z^{2}+3z^{4}\right)}{44}+\frac{\sqrt{462}Q_{xz}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{44}+\frac{\sqrt{462}Q_{yz}z\left(3x^{4}+6x^{2}y^{2}-8x^{2}z^{2}-2y^{2}z^{2}+z^{4}\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E_u,2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

** expression

$$-\frac{3\sqrt{22}Q_{u}y\left(5x^{4}+3x^{2}y^{2}-39x^{2}z^{2}-2y^{4}+17y^{2}z^{2}-2z^{4}\right)}{88}-\frac{\sqrt{66}Q_{v}y\left(9x^{4}-31x^{2}y^{2}+39x^{2}z^{2}+2y^{4}+11y^{2}z^{2}-12z^{4}\right)}{88}+\frac{\sqrt{66}Q_{xy}x\left(x^{4}-12x^{2}y^{2}+2x^{2}z^{2}+8y^{4}-12y^{2}z^{2}+z^{4}\right)}{44}+\frac{21\sqrt{66}Q_{xz}xyz\left(x-z\right)\left(x+z\right)}{44}-\frac{\sqrt{66}Q_{yz}z\left(x^{4}-12x^{2}y^{2}+2x^{2}z^{2}+8y^{4}-12y^{2}z^{2}+z^{4}\right)}{44}$$

$$-\frac{3\sqrt{22}Q_{u}x\left(2x^{4}-3x^{2}y^{2}-17x^{2}z^{2}-5y^{4}+39y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{66}Q_{v}x\left(2x^{4}-31x^{2}y^{2}+11x^{2}z^{2}+9y^{4}+39y^{2}z^{2}-12z^{4}\right)}{88}-\frac{\sqrt{66}Q_{xy}y\left(8x^{4}-12x^{2}y^{2}-12x^{2}z^{2}+y^{4}+2y^{2}z^{2}+z^{4}\right)}{44}+\frac{\sqrt{66}Q_{xz}z\left(8x^{4}-12x^{2}y^{2}-12x^{2}z^{2}+y^{4}+2y^{2}z^{2}+z^{4}\right)}{44}-\frac{21\sqrt{66}Q_{yz}xyz\left(y-z\right)\left(y+z\right)}{44}$$