PG No. 3 C_2 2 (b-axis setting) [monoclinic] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1
$\vec{\mathbb{G}}_1^{(1,0)}[q](A)$
** symmetry

$$y$$

** expression

$$-\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B,1)$
** symmetry

$$x$$

** expression

$$\frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B,2)$
** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2}$$

* Harmonics for rank 2
$\vec{\mathbb{G}}_2^{(1,0)}[q](A,1)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_x yz}{2} - \frac{\sqrt{6}Q_y xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A,2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x yz}{2} + \frac{\sqrt{2}Q_y xz}{2} - \sqrt{2}Q_z xy$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A,3)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x xy}{2} - \frac{\sqrt{2}Q_y\,(x-z)\,(x+z)}{2} - \frac{\sqrt{2}Q_z yz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B,1)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_x\,(y-z)\,(y+z)}{2} - \frac{\sqrt{2}Q_y xy}{2} + \frac{\sqrt{2}Q_z xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B,2)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{2}Q_x xz}{2} + \frac{\sqrt{2}Q_y yz}{2} + \frac{\sqrt{2}Q_z (x-y)(x+y)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,0)}[q](A,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_x x (y-z)(y+z)}{2} - \frac{\sqrt{5}Q_y y (x-z)(x+z)}{2} + \frac{\sqrt{5}Q_z z (x-y)(x+y)}{2}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A,2)$

** symmetry

$$-\frac{y\left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_x z \left(x^2 - 4y^2 + z^2\right)}{4} - \frac{\sqrt{3}Q_z x \left(x^2 - 4y^2 + z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A,3)$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x z \left(x^2 + 2y^2 - z^2\right)}{4} - \sqrt{5}Q_y xyz - \frac{\sqrt{5}Q_z x \left(x^2 - 2y^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,1)$

** symmetry

$$\frac{x\left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_y z \left(4x^2 - y^2 - z^2\right)}{4} - \frac{\sqrt{3}Q_z y \left(4x^2 - y^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,2)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y \left(x^2 + y^2 - 4z^2\right)}{4} + \frac{\sqrt{3}Q_y x \left(x^2 + y^2 - 4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,3)$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-\sqrt{5}Q_x xyz + \frac{\sqrt{5}Q_y z \left(2x^2 + y^2 - z^2\right)}{4} + \frac{\sqrt{5}Q_z y \left(2x^2 - y^2 + z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,4)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x y \left(x^2 - y^2 + 2z^2\right)}{4} - \frac{\sqrt{5}Q_y x \left(x^2 - y^2 - 2z^2\right)}{4} - \sqrt{5}Q_z xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_xyz\,(y-z)\,(y+z)}{6} + \frac{\sqrt{105}Q_yxz\,(x-z)\,(x+z)}{6} - \frac{\sqrt{105}Q_zxy\,(x-y)\,(x+y)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{3}Q_xyz\left(9x^2 + 2y^2 - 5z^2\right)}{6} + \frac{\sqrt{3}Q_yxz\left(2x^2 + 9y^2 - 5z^2\right)}{6} + \frac{7\sqrt{3}Q_zxy\,(x-y)\,(x+y)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$-\frac{Q_xyz\left(3x^2 - 4y^2 + 3z^2\right)}{2} + \frac{Q_yxz\left(4x^2 - 3y^2 - 3z^2\right)}{2} - \frac{Q_zxy\left(x^2 + y^2 - 6z^2\right)}{2}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_xxy\left(x^2 - 3z^2\right)}{4} + \frac{\sqrt{7}Q_y\left(x^2 - 2xz - z^2\right)\left(x^2 + 2xz - z^2\right)}{4} + \frac{\sqrt{7}Q_zyz\left(3x^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$-\frac{Q_xxy\left(x^2 - 6y^2 + 15z^2\right)}{4} + \frac{Q_y\,(x-z)\,(x+z)\left(x^2 - 6y^2 + z^2\right)}{4} + \frac{Q_zyz\left(15x^2 - 6y^2 + z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,1)$

** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x\left(y^2 - 2yz - z^2\right)\left(y^2 + 2yz - z^2\right)}{4} - \frac{\sqrt{7}Q_yxy\left(y^2 - 3z^2\right)}{4} + \frac{\sqrt{7}Q_zxz\left(3y^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,2)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_xxz\left(x^2 - 3y^2\right)}{4} + \frac{\sqrt{7}Q_yyz\left(3x^2 - y^2\right)}{4} + \frac{\sqrt{7}Q_z\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,3)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2 - y^2 - z^2\right)}{2}$$

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** expression

$$\frac{Q_x\,(y-z)\,(y+z)\,\left(6x^2-y^2-z^2\right)}{4}-\frac{Q_y xy\,\left(6x^2-y^2-15z^2\right)}{4}+\frac{Q_z xz\,\left(6x^2-15y^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,4)$
** symmetry

$$-\frac{\sqrt{5}xy\,\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{Q_x xz\,\left(x^2+15y^2-6z^2\right)}{4}-\frac{Q_y yz\,\left(15x^2+y^2-6z^2\right)}{4}-\frac{Q_z\,(x-y)\,(x+y)\,\left(x^2+y^2-6z^2\right)}{4}$$