

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag                            | matrix (polar)   | det | TR |
|-----|--------------------------------|--|-----|----|
| 1   | $\{1 0\}$                      | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$              | 1   | 1  |
| 2   | $\{3_{001}^+ 0\}$              | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | 1   | 1  |
| 3   | $\{3_{001}^- 0\}$              | $\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | 1   | 1  |
| 4   | $\{2_{100} 0\}$                | $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$           | 1   | 1  |
| 5   | $\{2_{110} 0\}$                | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$             | 1   | 1  |
| 6   | $\{2_{010} 0\}$                | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$           | 1   | 1  |
| 7   | $\{-6_{001}^+ 00\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | 1  |
| 8   | $\{m_{001} 00\frac{1}{2}\}$    | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$   | -1  | 1  |
| 9   | $\{-6_{001}^- 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | 1  |
| 10  | $\{m_{210} 00\frac{1}{2}\}$    | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$  | -1  | 1  |
| 11  | $\{m_{120} 00\frac{1}{2}\}$    | $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$  | -1  | 1  |
| 12  | $\{m_{1-10} 00\frac{1}{2}\}$   | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | -1  | 1  |
| 13  | $\{1' 0\}$                     | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$              | 1   | -1 |

continued ...

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|-----|--------------------------------|--|-----|----|
| 14  | $\{3_{001}^+ 0\}$              | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | 1   | -1 |
| 15  | $\{3_{001}^- 0\}$              | $\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$            | 1   | -1 |
| 16  | $\{2_{100}' 0\}$               | $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$           | 1   | -1 |
| 17  | $\{2_{110}' 0\}$               | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$             | 1   | -1 |
| 18  | $\{2_{010}' 0\}$               | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$           | 1   | -1 |
| 19  | $\{-6_{001}^+ 00\frac{1}{2}\}$ | $\begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 20  | $\{m_{001}' 00\frac{1}{2}\}$   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$   | -1  | -1 |
| 21  | $\{-6_{001}^- 00\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$ | -1  | -1 |
| 22  | $\{m_{210}' 00\frac{1}{2}\}$   | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$  | -1  | -1 |
| 23  | $\{m_{120}' 00\frac{1}{2}\}$   | $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$  | -1  | -1 |
| 24  | $\{m_{1-10}' 00\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$    | -1  | -1 |